

Matrix matrix multiplication:

Suppose there are two matrices A, B with dimensions as $m \times n$ and $n \times p$ respectively. Now we know that the result will be another matrix C with dimension $m \times p$.

For the sequential method the complexity of the method to compute the multiplication will be $O(n^3)$ that is order of mnp .

Method:

```
loop i from 0 to m-1
  loop j from 0 to p - 1
    loop k from 0 to n-1
      compute C[i][j] += A[i][k]*B[k][j];
    end loop
  end loop
end loop
```

parallelization:

To parallelize the multiplication of two matrices we need to apply the concept of matrix vector multiplication. In matrix vector multiplication the A matrix can be visualized as a collection of m vectors of size n and in the end matrix vector multiplication is reduced to simultaneous m dot product between the vectors.

$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 + 1 \cdot 1 + 1 \cdot 2 \\ 2 \cdot 3 + 1 \cdot 1 + 3 \cdot 2 \\ 1 \cdot 3 + 4 \cdot 1 + 2 \cdot 2 \end{pmatrix} \quad \text{First row,}$$
$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 + 1 \cdot 1 + 1 \cdot 2 \\ 2 \cdot 3 + 1 \cdot 1 + 3 \cdot 2 \\ 1 \cdot 3 + 4 \cdot 1 + 2 \cdot 2 \end{pmatrix} \quad \text{next row,}$$
$$\begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 4 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \cdot 3 + 1 \cdot 1 + 1 \cdot 2 \\ 2 \cdot 3 + 1 \cdot 1 + 3 \cdot 2 \\ 1 \cdot 3 + 4 \cdot 1 + 2 \cdot 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 13 \\ 11 \end{pmatrix} \quad \text{last row, then do the addition.}$$

So, Now in matrix matrix multiplication we can think of the B matrix as a collection of column vectors. And similarly with the matrix vector multiplication we are basically calculating the dot product of different vectors to generate the result.

So row vectors of the first matrix and the column vectors of the second matrix is responsible for the result of the matrix matrix multiplication in the following way:

- The i, j th position of the result matrix that is C. Is nothing but the dot product of i th row vector of the A matrix and j th column vector of B matrix.

So, one of the methodology will be

- At first transpose the B matrix creating a collection of row vectors for the ease of access, creating a $p \times n$ matrix as B had dimension of $n \times p$.
- Now pass part of the both matrix to different processes. Say there are P processes, so pass m/p number of rows from the A matrix and p/P number of rows from the B matrix to each process.
- Compute the local dot product of row vectors of each submatrix. The dot product between i th row vector of local submatrix of A and j th row of local submatrix of B create the result of the (i,j) th position of C matrix.

